# Package 'fromo'

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Type Package
Maintainer Steven E. Pav <shabbychef@gmail.com>
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Title Fast Robust Moments
BugReports https://github.com/shabbychef/fromo/issues
Description Fast, numerically robust computation of weighted moments via 'Rcpp'.
Supports computation on vectors and matrices, and Monoidal append of moments.
Moments and cumulants over running fixed length windows can be computed,
as well as over time-based windows.
Moment computations are via a generalization of Welford's method, as described
by Bennett et. (2009) <doi:10.1109/CLUSTR.2009.5289161>.

Imports Rcpp (>= 0.12.3), methods

LinkingTo Rcpp

Suggests knitr, testthat, moments, PDQutils, e1071, microbenchmark

RoxygenNote 7.3.2

URL https://github.com/shabbychef/fromo

VignetteBuilder knitr

Collate 'fromo.r' 'RcppExports.R' 'zzz\_centsums.R'

NeedsCompilation yes

Author Steven E. Pav [aut, cre] (ORCID: <a href="https://orcid.org/0000-0002-4197-6195">https://orcid.org/0000-0002-4197-6195</a>)

**Repository** CRAN

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fromo-package

Fast Robust Moments.

### Description

Fast, numerically robust moments computations, along with computation of cumulants, running means, etc.

# **Robust Moments**

Welford described a method for 'robust' one-pass computation of the standard deviation. By 'robust', we mean robust to round-off caused by a large shift in the mean. This method was generalized by Terriberry, and Bennett *et. al.* to the case of higher-order moments. This package provides those algorithms for computing moments.

Generally we should find that the stock implementations of sd, skewness and so on are *already* robust and likely using these algorithms under the hood. This package was written for a few reasons:

#### fromo-package

- 1. As an exercise to learn Rcpp.
- 2. Often I found I needed the first k moments. For example, when computing the Z-score, the standard deviation and mean must be computed separately, which is inefficient. Similarly Merten's correction for the standard error of the Sharpe ratio uses the first four moments. These are all computed as a side effect of computation of the kurtosis, but discarded by the standard methods.

### Legal Mumbo Jumbo

fromo is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PUR-POSE. See the GNU Lesser General Public License for more details.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

This package was developed as an exercise in learning Rcpp.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

Maintainer: Steven E. Pav <shabbychef@gmail.com> (ORCID)

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

# See Also

Useful links:

- https://github.com/shabbychef/fromo
- Report bugs at https://github.com/shabbychef/fromo/issues

accessor

### Description

Access slot data from a centsums object.

### Usage

sums(x)
## S4 method for signature 'centsums'
sums(x)
moments(x, type = c("central", "raw", "standardized"))
## S4 method for signature 'centsums'
moments(x, type = c("central", "raw", "standardized"))

# Arguments

Х	a centsums object.
type	the type of moment to compute.

### Note

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### Author(s)

Steven E. Pav <shabbychef@gmail.com>

as.centcosums Coerce to a centcosums object.

# Description

Convert data to a centcosums object.

#### Usage

```
as.centcosums(x, order=2, na.omit=TRUE)
## Default S3 method:
as.centcosums(x, order = 2, na.omit = TRUE)
```

### as.centsums

#### Arguments

х	a matrix.
order	the order, defaulting to 2.
na.omit	whether to remove rows with NA.

# Details

Computes the raw cosums on data, and stuffs the results into a centcosums object.

### Value

A centcosums object.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### Examples

set.seed(123)
x <- matrix(rnorm(100\*3),ncol=3)
cs <- as.centcosums(x, order=2)</pre>

as.centsums

Coerce to a centsums object.

### Description

Convert data to a centsums object.

### Usage

```
as.centsums(
    x,
    order = 3,
    na.rm = TRUE,
    wts = NULL,
    check_wts = FALSE,
    normalize_wts = FALSE
)
```

```
## Default S3 method:
as.centsums(
    x,
    order = 3,
    na.rm = TRUE,
    wts = NULL,
    check_wts = FALSE,
    normalize_wts = FALSE
)
```

# Arguments

x	a numeric, array, or matrix.
order	the order, defaulting to length(sums)+1.
na.rm	whether to remove NA.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding $v$ value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking $v$ are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

# Details

Computes the raw sums on data, and stuffs the results into a centsums object.

### Value

A centsums object.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

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# c.centcosums

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

# Examples

```
set.seed(123)
x <- rnorm(1000)
cs <- as.centsums(x, order=5)</pre>
```

c.centcosums concatenate centcosums objects.

# Description

Concatenate centcosums objects.

# Usage

\method{c}{centcosums}(...)

# Arguments

... centcosums objects

### See Also

join\_cent\_cosums

c.centsums

concatenate centsums objects.

# Description

Concatenate centsums objects.

# Usage

\method{c}{centsums}(...)

### Arguments

... centsums objects

### See Also

join\_cent\_sums

cent2raw

### Description

Given raw or central or standardized moments, convert to another type.

### Usage

```
cent2raw(input)
```

### Arguments

input

a vector of the count, then the mean, then the 2 through  $k\ raw$  or central moments.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

centcosums-accessor Accessor methods.

### Description

Access slot data from a centcosums object.

### Usage

```
cosums(x)
```

```
## S4 method for signature 'centcosums'
cosums(x)
comoments(x, type = c("central", "raw"))
## S4 method for signature 'centcosums'
comoments(x, type = c("central", "raw"))
```

### centcosums-class

#### Arguments

х	a centcosums object.
type	the type of moment to compute.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

lass.
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### Description

An S4 class to store (centered) cosums of data, and to support operations on the same.

### Usage

```
## S4 method for signature 'centcosums'
initialize(.Object, cosums, order = NA_real_)
```

```
centcosums(cosums, order = NULL)
```

### Arguments

.Object	a centcosums object, or proto-object.
cosums	the output of cent_cosums, say.
order	the order, defaulting to 2.

# Details

A centcosums object contains a multidimensional array (now only 2-diemnsional), as output by cent\_cosums.

# Value

An object of class centcosums.

# Slots

cosums a multidimensional array of the cosums. order the maximum order. ignored for now. The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

### See Also

cent\_cosums

#### Examples

obj <- new("centcosums", cosums=cent\_cosums(matrix(rnorm(100\*3), ncol=3), max\_order=2), order=2)

centsums-class centsums Class.

#### Description

An S4 class to store (centered) sums of data, and to support operations on the same.

### Usage

```
## S4 method for signature 'centsums'
initialize(.Object, sums, order = NA_real_)
```

```
centsums(sums, order = NULL)
```

### Arguments

.Object	a centsums object, or proto-object.
sums	a numeric vector.
order	the order, defaulting to length(sums)+1.

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#### centsums-class

### Details

A centsums object contains a vector value of the data count, the mean, and the kth centered sum, for k up to some maximum order.

#### Value

An object of class centsums.

### Slots

sums a numeric vector of the sums.

order the maximum order.

### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

### Examples

obj <- new("centsums",sums=c(1000,1.234,0.235),order=2)</pre>

cent\_cosums

# Description

Compute, join, or unjoin multivariate centered (co-) sums.

#### Usage

```
cent_cosums(v, max_order = 2L, na_omit = FALSE)
cent_comoments(v, max_order = 2L, used_df = 0L, na_omit = FALSE)
join_cent_cosums(ret1, ret2)
```

```
unjoin_cent_cosums(ret3, ret2)
```

# Arguments

V	an $m$ by $n$ matrix, each row an independent observation of some $n$ variate variable.
max_order	the maximum order of cosum to compute. For now this can only be 2; in the future higher order cosums should be possible.
na_omit	a boolean; if TRUE, then only rows of $\boldsymbol{v}$ with complete observations will be used.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
ret1	a multdimensional array as output by cent_cosums.
ret2	a multdimensional array as output by cent_cosums.
ret3	a multdimensional array as output by cent_cosums.

### Value

a multidimensional arry of dimension max\_order, each side of length 1 + n. For the case currently implemented where max\_order must be 2, the output is a symmetric matrix, where the element in the 1,1 position is the count of complete) rows of v, the 2:(n+1),1 column is the mean, and the 2:(n+1),2:(n+1) is the co *sums* matrix, which is the covariance up to scaling by the count. cent\_comments performs this normalization for you.

### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

### cent\_sums

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

# References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

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#### See Also

cent\_sums

#### Examples

```
set.seed(1234)
x1 <- matrix(rnorm(1e3*5,mean=1),ncol=5)
x2 <- matrix(rnorm(1e3*5,mean=1),ncol=5)
max_ord <- 2L
rs1 <- cent_cosums(x1,max_ord)
rs2 <- cent_cosums(x2,max_ord)
rs3 <- cent_cosums(rbind(x1,x2),max_ord)
rs3alt <- join_cent_cosums(rs1,rs2)
stopifnot(max(abs(rs3 - rs3alt)) < 1e-7)
rs1alt <- unjoin_cent_cosums(rs3,rs1)
stopifnot(max(abs(rs1 - rs1alt)) < 1e-7)
stopifnot(max(abs(rs2 - rs2alt)) < 1e-7)</pre>
```

cent\_sums

Centered sums; join and unjoined.

# Description

Compute, join, or unjoin centered sums.

# Usage

```
cent_sums(
    v,
    max_order = 5L,
    na_rm = FALSE,
    wts = NULL,
    check_wts = FALSE,
    normalize_wts = TRUE
)
join_cent_sums(ret1, ret2)
unjoin_cent_sums(ret3, ret2)
```

# Arguments

v	a vector
max_order	the maximum order of the centered moment to be computed.
na_rm	whether to remove NA, false by default.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.
ret1	an $ord + 1$ vector as output by cent_sums consisting of the count, the mean, then the k through ordth centered sum of some observations.
ret2	an $ord + 1$ vector as output by cent_sums consisting of the count, the mean, then the k through ordth centered sum of some observations.
ret3	an $ord + 1$ vector as output by cent_sums consisting of the count, the mean, then the k through ordth centered sum of some observations.

# Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

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### fromo-NEWS

# Note

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#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

### Examples

```
set.seed(1234)
x1 <- rnorm(1e3,mean=1)
x2 <- rnorm(1e3,mean=1)
max_ord <- 6L
rs1 <- cent_sums(x1,max_ord)
rs2 <- cent_sums(x2,max_ord)
rs3 <- cent_sums(c(x1,x2),max_ord)
rs3alt <- join_cent_sums(rs1,rs2)
stopifnot(max(abs(rs3 - rs3alt)) < 1e-7)
rs1alt <- unjoin_cent_sums(rs3,rs1)
stopifnot(max(abs(rs1 - rs1alt)) < 1e-7)
stopifnot(max(abs(rs2 - rs2alt)) < 1e-7)</pre>
```

fromo-NEWS

News for package 'fromo':

### Description

News for package 'fromo'

### Version 0.2.4 (2024-11-29)

- adding running correlation, covariance, regression coefficients.
- adding t-running correlation, covariance, regression coefficients.

### Version 0.2.3 (2024-11-07)

- adding flag and checking for negative even moments due to roundoff.
- fixing divide by zero which would corrupt results when input was constant.

### Version 0.2.2 (2024-11-03)

• fix t\_running\_sum and others to act as documented when variable\_win is flagged.

### Version 0.2.1 (2019-01-29)

• fix memory leak for case where the mean only need be computed via a Welford object.

### Version 0.2.0 (2019-01-12)

- add std\_cumulants
- add running\_sum, running\_mean.
- Kahan compensated summation for these.
- Welford object under the hood.
- add weighted moments computation.
- add time-based running window computations.
- some speedups for obviously fast cases: no checking of NA, etc.
- move github figures to location CRAN understands.

### Version 0.1.3 (2016-04-04)

· submit to CRAN

### Initial Version 0.1.0 (2016-03-25)

· start work

running\_apx\_quantiles Compute approximate quantiles over a sliding window

# Description

Computes cumulants up to some given order, then employs the Cornish-Fisher approximation to compute approximate quantiles using a Gaussian basis.

# Usage

```
running_apx_quantiles(
 ν,
 p,
 window = NULL,
 wts = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
 restart_period = 100L,
 check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_apx_median(
  ν,
 window = NULL,
 wts = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
 restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

# Arguments

v	a vector
р	the probability points at which to compute the quantiles. Should be in the range $(0,1)$ .
window	the window size. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the

weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

max\_order the maximum order of the centered moment to be computed.

- na\_rm whether to remove NA, false by default.
- min\_df the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
- used\_df the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
- restart\_period the recompute period. because subtraction of elements can cause loss of precision, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.
- check\_wts a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
- normalize\_wts a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

### check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

#### Details

Computes the cumulants, then approximates quantiles using AS269 of Lee & Lin.

### Value

A matrix, with one row for each element of x, and one column for each element of q.

### Note

The current implementation is not as space-efficient as it could be, as it first computes the cumulants for each row, then performs the Cornish-Fisher approximation on a row-by-row basis. In the future, this computation may be moved earlier into the pipeline to be more space efficient. File an issue if the memory footprint is an issue for you.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

### See Also

t\_running\_apx\_quantiles, running\_cumulants, PDQutils::qapx\_cf, PDQutils::AS269.

# Examples

```
x <- rnorm(1e5)
xq <- running_apx_quantiles(x,c(0.1,0.25,0.5,0.75,0.9))
xm <- running_apx_median(x)</pre>
```

running\_centered Compare data to moments computed over a sliding window.

### Description

Computes moments over a sliding window, then adjusts the data accordingly, centering, or scaling, or z-scoring, and so on.

# Usage

```
running_centered(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = FALSE,
  check_negative_moments = TRUE
)
running_scaled(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_zscored(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
 lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_sharpe(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
```

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# running\_centered

```
compute_se = FALSE,
 min_df = 0L,
 used_df = 1,
 restart_period = 100L,
 check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
running_tstat(
 ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
 restart_period = 100L,
 check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
```

# Arguments

V	a vector
window	the window size. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent <i>e.g.</i> Z-scores from being computed on only 3 observations. Defaults to zero, meaning no restriction, which can result in infinite Z-scores during the burn-in period.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.

lookahead for some of the operations, the value is compared to mean and standard deviation possibly using 'future' or 'past' information by means of a non-zero lookahead. Positive values mean data are taken from the future. restart\_period the recompute period. because subtraction of elements can cause loss of precision, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results. a boolean for whether the code shall check for negative weights, and throw an check wts error when they are found. Default false for speed. a boolean for whether the weights should be renormalized to have a mean value normalize\_wts of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector. check\_negative\_moments a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every

even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

for running\_sharpe, return an extra column of the standard error, as computed compute\_se by Mertens' correction.

#### Details

Given the length n vector x, for a given index i, define  $x^{(i)}$  as the vector of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ , where we do not run over the 'edge' of the vector. In code, this is essentially x[(max(1, i-window+1)): i]. Then define  $\mu_i$ ,  $\sigma_i$  and  $n_i$  as, respectively, the sample mean, standard deviation and number of non-NA elements in  $x^{(i)}$ .

We compute output vector m the same size as x. For the 'centered' version of x, we have  $m_i =$  $x_i - \mu_i$ . For the 'scaled' version of x, we have  $m_i = x_i/\sigma_i$ . For the 'z-scored' version of x, we have  $m_i = (x_i - \mu_i)/\sigma_i$ . For the 't-scored' version of x, we have  $m_i = \sqrt{n_i}\mu_i/\sigma_i$ .

We also allow a 'lookahead' for some of these operations. If positive, the moments are computed using data from larger indices; if negative, from smaller indices. Letting j = i + lookahead: For the 'centered' version of x, we have  $m_i = x_i - \mu_i$ . For the 'scaled' version of x, we have  $m_i = x_i/\sigma_i$ . For the 'z-scored' version of x, we have  $m_i = (x_i - \mu_j)/\sigma_j$ .

### Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

### See Also

t\_running\_centered, scale

### Examples

```
if (require(moments)) {
    set.seed(123)
    x <- rnorm(5e1)
   window <- 10L
    rm1 <- t(sapply(seq_len(length(x)),function(iii) {</pre>
                   xrang <- x[max(1,iii-window+1):iii]</pre>
                   c(sd(xrang),mean(xrang),length(xrang)) },
                   simplify=TRUE))
    rcent <- running_centered(x,window=window)</pre>
    rscal <- running_scaled(x,window=window)</pre>
    rzsco <- running_zscored(x,window=window)</pre>
    rshrp <- running_sharpe(x,window=window)</pre>
    rtsco <- running_tstat(x,window=window)</pre>
    rsrse <- running_sharpe(x,window=window,compute_se=TRUE)</pre>
    stopifnot(max(abs(rcent - (x - rm1[,2])),na.rm=TRUE) < 1e-12)</pre>
    stopifnot(max(abs(rscal - (x / rm1[,1])),na.rm=TRUE) < 1e-12)</pre>
    stopifnot(max(abs(rzsco - ((x - rm1[,2]) / rm1[,1])),na.rm=TRUE) < 1e-12)</pre>
    stopifnot(max(abs(rshrp - (rm1[,2] / rm1[,1])),na.rm=TRUE) < 1e-12)</pre>
```

running\_correlation Compute covariance, correlation, regression over a sliding window

# Description

}

Computes 2nd moments and comoments, as well as the means, over an infinite or finite sliding window, returning a matrix with the correlation, covariance, regression coefficient, and so on.

### Usage

```
running_correlation(
  х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 100L,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
running_covariance(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

```
running_covariance_3(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_regression_slope(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 100L,
 check_wts = FALSE,
  check_negative_moments = TRUE
)
running_regression_intercept(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 check_negative_moments = TRUE
)
running_regression_fit(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  check_wts = FALSE,
  check_negative_moments = TRUE
```

```
running_regression_diagnostics(
    X,
    y,
    window = NULL,
    wts = NULL,
    na_rm = FALSE,
    min_df = 0L,
    used_df = 2,
    restart_period = 100L,
    check_wts = FALSE,
    normalize_wts = TRUE,
    check_negative_moments = TRUE
)
```

# Arguments

x	a vector	
У	a vector	
window	the window size. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.	
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.	
na_rm	whether to remove NA, false by default.	
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.	
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this param- eter. Larger values mean fewer restarts and faster, though less accurate results.	
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.	
check_negative_moments		
	a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this	

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)

	flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.
used_df	the number of degrees of freedom consumed, used in the denominator of the standard errors computation. These are subtracted from the number of observa- tions.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

### Details

Computes the correlation or covariance, or OLS regression coefficients and standard errors. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

#### Value

Typically a matrix, usually only one row of the output value. More specifically:

running\_covariance Returns a single column of the covariance of x and y.

running\_correlation Returns a single column of the correlation of x and y.

- **running\_covariance\_3** Returns three columns: the variance of x, the covariance of x and y, and the variance of y, in that order.
- running\_regression\_slope Returns a single column of the slope of the OLS regression.
- running\_regression\_intercept Returns a single column of the intercept of the OLS regression.
- **running\_regression\_fit** Returns two columns: the regression intercept and the regression slope of the OLS regression.
- **running\_regression\_diagnostics** Returns five columns: the regression intercept, the regression slope, the regression standard error, the standard error of the intercept, the standard error of the slope of the OLS regression.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

#### Examples

```
x <- rnorm(1e5)
y <- rnorm(1e5) + x
rho <- running_correlation(x, y, window=100L)</pre>
```

running\_sd3

Compute first K moments over a sliding window

# Description

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements over an infinite or finite sliding window, returning a matrix.

### Usage

```
running_sd3(
    v,
    window = NULL,
    wts = NULL,
    na_rm = FALSE,
    min_df = 0L,
    used_df = 1,
    restart_period = 100L,
    check_wts = FALSE,
    normalize_wts = TRUE,
```

```
check_negative_moments = TRUE
)
running_skew4(
 ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
 restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
running_kurt5(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
 check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_sd(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
running_skew(
 v,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
```

```
used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_kurt(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_cent_moments(
 v,
 window = NULL,
 wts = NULL,
 max_order = 5L,
 na_rm = FALSE,
 max_order_only = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_std_moments(
 ν,
 window = NULL,
 wts = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

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```
running_cumulants(
    v,
    window = NULL,
    wts = NULL,
    max_order = 5L,
    na_rm = FALSE,
    min_df = 0L,
    used_df = 0,
    restart_period = 100L,
    check_wts = FALSE,
    normalize_wts = TRUE,
    check_negative_moments = TRUE
)
```

# Arguments

v	a vector
window	the window size. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this param- eter. Larger values mean fewer restarts and faster, though less accurate results.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so

if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

max\_order the maximum order of the centered moment to be computed.

max\_order\_only for running\_cent\_moments, if this flag is set, only compute the maximum order centered moment, and return in a vector.

### Details

Computes the number of elements, the mean, and the 2nd through kth centered (and typically standardized) moments, for k = 2, 3, 4. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

Given the length *n* vector *x*, we output matrix *M* where  $M_{i,j}$  is the order-j+1 moment (*i.e.* excess kurtosis, skewness, standard deviation, mean or number of elements) of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ . Barring NA or NaN, this is over a window of size window. During the 'burn-in' phase, we take fewer elements.

# Value

Typically a matrix, where the first columns are the kth, k-1th through 2nd standardized, centered moments, then a column of the mean, then a column of the number of (non-nan) elements in the input, with the following exceptions:

- **running\_cent\_moments** Computes arbitrary order centered moments. When max\_order\_only is set, only a column of the maximum order centered moment is returned.
- **running\_std\_moments** Computes arbitrary order standardized moments, then the standard deviation, the mean, and the count. There is not yet an option for max\_order\_only, but probably should be.
- **running\_cumulants** Computes arbitrary order cumulants, and returns the kth, k-1th, through the second (which is the variance) cumulant, then the mean, and the count.

#### Note

the kurtosis is excess kurtosis, with a 3 subtracted, and should be nearly zero for Gaussian input.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### running\_sd3

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

# Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

#### Examples

xc6 <- running\_cent\_moments(x,window=100L,max\_order=6L)</pre>

running\_sum

# Description

Compute the mean or sum over an infinite or finite sliding window, returning a vector the same size as the input.

# Usage

```
running_sum(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 restart_period = 10000L,
 check_wts = FALSE
)
running_mean(
  ν,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 10000L,
  check_wts = FALSE
)
```

# Arguments

ν	a vector.
window	the window size. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.

#### running\_sum

na_rm	whether to remove NA, false by default.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this parame- ter. Larger values mean fewer restarts and faster, though potentially less accurate results. Unlike in the computation of even order moments, loss of precision is unlikely to be disastrous, so the default value is rather large.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
min_df	the minimum df to return a value, otherwise NaN is returned, only for the means computation. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.

#### Details

Computes the mean or sum of the elements, using a Kahan's Compensated Summation Algorithm, a numerically robust one-pass method.

Given the length n vector x, we output matrix M where  $M_{i,1}$  is the sum or mean of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ . Barring NA or NaN, this is over a window of size window. During the 'burn-in' phase, we take fewer elements. If fewer than min\_df for running\_mean, returns NA.

### Value

A vector the same size as the input.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

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# Examples

x <- rnorm(1e5)
xs <- running\_sum(x,10)
xm <- running\_mean(x,100)</pre>

sd3

#### Compute first K moments

# Description

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements.

# Usage

```
sd3(
  ٧,
  na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize_wts = TRUE
)
skew4(
  ν,
 na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize_wts = TRUE
)
kurt5(
  ν,
  na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize_wts = TRUE
)
cent_moments(
```

36

37

```
ν,
 max_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
  normalize_wts = TRUE
)
std_moments(
  ν,
 max_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
 normalize_wts = TRUE
)
cent_cumulants(
 ν,
 max_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
 normalize_wts = TRUE
)
std_cumulants(
  ν,
 max_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
 check_wts = FALSE,
 normalize_wts = TRUE
)
```

# Arguments

V	a vector
na_rm	whether to remove NA, false by default.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check

sd3

	for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
sg_df	the number of degrees of freedom consumed in the computation of the variance or standard deviation. This defaults to 1 to match the 'Bessel correction'.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.
max_order	the maximum order of the centered moment to be computed.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.

### Details

Computes the number of elements, the mean, and the 2nd through kth centered standardized moment, for k = 2, 3, 4. These are computed via the numerically robust one-pass method of Bennett *et. al.* In general they will *not* match exactly with the 'standard' implementations, due to differences in roundoff.

These methods are reasonably fast, on par with the 'standard' implementations. However, they will usually be faster than calling the various standard implementations more than once.

Moments are computed as follows, given some values  $x_i$  and optional weights  $w_i$ , defaulting to 1, the weighted mean is computed as

$$\mu = \frac{\sum_i x_i w_i}{\sum w_i}.$$

The weighted kth central sum is computed as

$$S_k = \sum_i \left( x_i - \mu \right)^k w_i.$$

Let  $n = \sum_{i} w_i$  be the sum of weights (or number of observations in the unweighted case). Then the weighted kth central moment is computed as that weighted sum divided by the adjusted sum weights:

$$\mu_k = \frac{\sum_i \left(x_i - \mu\right)^k w_i}{n - \nu},$$

where  $\nu$  is the 'used df', provided by the user to adjust the denominator. (Typical values are 0 or 1.) The weighted kth standardized moment is the central moment divided by the second central moment to the k/2 power:

$$\tilde{\mu}_k = \frac{\mu_k}{\mu_2^{k/2}}.$$

The (centered) rth cumulant, for  $r \ge 2$  is then computed using the formula of Willink, namely

$$\kappa_r = \mu_r - \sum_{j=1}^{r-2} \binom{r-1}{j} \kappa_{r-j} \mu_j,$$

That is

$$\begin{aligned} \kappa_2 &= \mu_2, \\ \kappa_3 &= \mu_3, \\ \kappa_4 &= \mu_4 - 3\mu_2^2, \\ \kappa_5 &= \mu_5 - 10\mu_3\mu_2, \end{aligned}$$

and so on.

The standardized rth cumulant is the rth centered cumulant divided by  $\mu_2^{r/2}$ .

#### Value

a vector, filled out as follows:

- sd3 A vector of the (sample) standard devation, mean, and number of elements (or the total weight when wts are given).
- **skew4** A vector of the (sample) skewness, standard devation, mean, and number of elements (or the total weight when wts are given).
- **kurt5** A vector of the (sample) excess kurtosis, skewness, standard devation, mean, and number of elements (or the total weight when wts are given).
- **cent\_moments** A vector of the (sample) kth centered moment, then k 1th centered moment, ..., then the *variance*, the mean, and number of elements (total weight when wts are given).
- **std\_moments** A vector of the (sample) kth standardized (and centered) moment, then k 1th, ..., then standard devation, mean, and number of elements (total weight).
- **cent\_cumulants** A vector of the (sample) kth (centered, but this is redundant) cumulant, then the k 1th, ..., then the *variance* (which is the second cumulant), then *the mean*, then the number of elements (total weight).
- **std\_cumulants** A vector of the (sample) *k*th standardized (and centered, but this is redundant) cumulant, then the k 1th, ..., down to the third, then *the variance*, *the mean*, then the number of elements (total weight).

#### Note

The first centered (and standardized) moment is often defined to be identically 0. Instead cent\_moments and std\_moments returns the mean. Similarly, the second standardized moments defined to be identically 1; std\_moments instead returns the standard deviation. The reason is that a user can always decide to ignore the results and fill in a 0 or 1 as they need, but could not efficiently compute the mean and standard deviation from scratch if we discard it. The antepenultimate element of the output of std\_cumulants is not a one, even though that 'should' be the standardized second cumulant.

The antepenultimate element of the output of cent\_moments, cent\_cumulants and std\_cumulants is the *variance*, not the standard deviation. All other code return the standard deviation in that place.

The kurtosis is excess kurtosis, with a 3 subtracted, and should be nearly zero for Gaussian input.

The term 'centered cumulants' is redundant. The intent was to avoid possible collision with existing code named 'cumulants'.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

# Author(s)

Steven E. Pav <shabbychef@gmail.com>

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#### Examples

```
x <- rnorm(1e5)
sd3(x)[1] - sd(x)
skew4(x)[4] - length(x)
skew4(x)[3] - mean(x)
skew4(x)[2] - sd(x)
if (require(moments)) {
    skew4(x)[1] - skewness(x)
}
# check 'robustness'; only the mean should change:
kurt5(x + 1e12) - kurt5(x)
# check speed
if (require(microbenchmark) && require(moments)) {
    dumbk <- function(x) { c(kurtosis(x) - 3.0,skewness(x),sd(x),mean(x),length(x)) }
    set.seed(1234)
```

show

```
x <- rnorm(1e6)</pre>
  print(kurt5(x) - dumbk(x))
  microbenchmark(dumbk(x),kurt5(x),times=10L)
}
y <- std_moments(x,6)</pre>
cml <- cent_cumulants(x,6)</pre>
std <- std_cumulants(x,6)</pre>
# check that skew matches moments::skewness
if (require(moments)) {
    set.seed(1234)
    x <- rnorm(1000)
    resu <- fromo::skew4(x)</pre>
    msku <- moments::skewness(x)</pre>
    stopifnot(abs(msku - resu[1]) < 1e-14)</pre>
}
# check skew vs e1071 skewness, which has a different denominator
if (require(e1071)) {
    set.seed(1234)
    x <- rnorm(1000)
    resu <- fromo::skew4(x)</pre>
    esku <- e1071::skewness(x,type=3)</pre>
    nobs <- resu[4]</pre>
    stopifnot(abs(esku - resu[1] * ((nobs-1)/nobs)^(3/2)) < 1e-14)</pre>
    # similarly:
    resu <- fromo::std_moments(x,max_order=3,used_df=0)</pre>
    stopifnot(abs(esku - resu[1] * ((nobs-1)/nobs)^(3/2)) < 1e-14)</pre>
}
```

show

Show a centsums object.

# Description

Displays the centsums object.

# Usage

show(object)

## S4 method for signature 'centsums'
show(object)

#### Arguments

object a centsums object.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

### Author(s)

Steven E. Pav <shabbychef@gmail.com>

# Examples

```
set.seed(123)
x <- rnorm(1000)
obj <- as.centsums(x, order=5)
obj</pre>
```

t\_running\_apx\_quantiles

Compute approximate quantiles over a sliding time window

# Description

Computes cumulants up to some given order, then employs the Cornish-Fisher approximation to compute approximate quantiles using a Gaussian basis.

# Usage

```
t_running_apx_quantiles(
  ν,
 р,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

```
t_running_apx_median(
 ν,
  time = NULL,
 time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
 restart_period = 100L,
 variable_win = FALSE,
 wts_as_delta = TRUE,
 check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
```

# Arguments

v	a vector of data.
р	the probability points at which to compute the quantiles. Should be in the range $(0,1)$ .
time	an optional vector of the timestamps of v. If given, must be the same length as v. If not given, we try to infer it by summing the time_deltas.
time_deltas	an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts_as_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.
window	the window size, in time units. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, and variable_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding $v$ value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking $v$ are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.

lb_time	a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.
max_order	the maximum order of the centered moment to be computed.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this param- eter. Larger values mean fewer restarts and faster, though less accurate results.
variable_win	if true, and the window is not a concrete number, the computation window becomes the time between lookback times.
wts_as_delta	if true and the time_deltas are not given, but wts are given, we take wts as the time_deltas.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.
check_negative_	moments
	a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

# Details

Computes the cumulants, then approximates quantiles using AS269 of Lee & Lin.

# Value

A matrix, with one row for each element of x, and one column for each element of q.

#### **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \ldots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of *lookback times* are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The *j*th output is computed over indices *i* such that

$$b_j - W < t_i \leq b_j$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the *j*th output is computed over indices *i* such that

$$t_i - W + L < t_i \le t_i + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

The current implementation is not as space-efficient as it could be, as it first computes the cumulants for each row, then performs the Cornish-Fisher approximation on a row-by-row basis. In the future, this computation may be moved earlier into the pipeline to be more space efficient. File an issue if the memory footprint is an issue for you.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

# See Also

running\_apx\_quantiles, t\_running\_cumulants, PDQutils::qapx\_cf, PDQutils::AS269.

#### Examples

t\_running\_centered Compare data to moments computed over a time sliding window.

#### Description

Computes moments over a sliding window, then adjusts the data accordingly, centering, or scaling, or z-scoring, and so on.

t\_running\_centered

# Usage

```
t_running_centered(
  ν,
  time = NULL,
  time_deltas = NULL,
  window = NULL,
 wts = NULL,
  na_rm = FALSE,
  min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_scaled(
  ν,
  time = NULL,
  time_deltas = NULL,
  window = NULL,
 wts = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_zscored(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
  wts = NULL,
  na_rm = FALSE,
  min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
```

```
variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_sharpe(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
  compute_se = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_tstat(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
  compute_se = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

# Arguments

v a vector of data.

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time	an optional vector of the timestamps of v. If given, must be the same length as v. If not given, we try to infer it by summing the time_deltas.
time_deltas	an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts_as_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.
window	the window size, in time units. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, and variable_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent <i>e.g.</i> Z-scores from being computed on only 3 observations. Defaults to zero, meaning no restriction, which can result in infinite Z-scores during the burn-in period.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
lookahead	for some of the operations, the value is compared to mean and standard deviation possibly using 'future' or 'past' information by means of a non-zero lookahead. Positive values mean data are taken from the future. This is in time units, and so should be a real.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this param- eter. Larger values mean fewer restarts and faster, though less accurate results.
variable_win	if true, and the window is not a concrete number, the computation window becomes the time between lookback times.
wts_as_delta	if true and the time_deltas are not given, but wts are given, we take wts as the time_deltas.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

normalize_wts	a boolean for whether the weights should be renormalized to have a mean value
	of 1. This mean is computed over elements which contribute to the moments, so
	if na_rm is set, that means non-NA elements of wts that correspond to non-NA
	elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

- lb\_time a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.
- compute\_se for running\_sharpe, return an extra column of the standard error, as computed by Mertens' correction.

#### Details

Given the length n vector x, for a given index i, define  $x^{(i)}$  as the elements of x defined by the sliding time window (see the section on time windowing). Then define  $\mu_i$ ,  $\sigma_i$  and  $n_i$  as, respectively, the sample mean, standard deviation and number of non-NA elements in  $x^{(i)}$ .

We compute output vector m the same size as x. For the 'centered' version of x, we have  $m_i = x_i - \mu_i$ . For the 'scaled' version of x, we have  $m_i = x_i/\sigma_i$ . For the 'z-scored' version of x, we have  $m_i = (x_i - \mu_i)/\sigma_i$ . For the 't-scored' version of x, we have  $m_i = \sqrt{n_i \mu_i}/\sigma_i$ .

We also allow a 'lookahead' for some of these operations. If positive, the moments are computed using data from larger indices; if negative, from smaller indices.

#### Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

#### **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \ldots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of *lookback times* are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The *j*th output is computed over indices *i* such that

$$b_j - W < t_i \le b_j.$$

#### t\_running\_centered

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the *j*th output is computed over indices *i* such that

$$t_j - W + L < t_i \le t_j + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

#### See Also

running\_centered, scale

t\_running\_correlation Compute covariance, correlation, regression over a sliding time-based window

# Description

Computes 2nd moments and comments, as well as the means, over an infinite or finite sliding time based window, returning a matrix with the correlation, covariance, regression coefficient, and so on.

#### Usage

```
t_running_correlation(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_covariance(
  х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

```
t_running_covariance_3(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_regression_slope(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_intercept(
 х,
 у,
 time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
```

```
wts_as_delta = TRUE,
 check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_fit(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
 variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_diagnostics(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
 used_df = 2,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

#### Arguments

Х	a vector
У	a vector
time	an optional vector of the timestamps of x and y. If given, must be the same length as x and y. If not given, we try to infer it by summing the time_deltas.
time_deltas	an optional vector of the deltas of timestamps. If given, must be the same length

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as x and y. If not given, and wts are given and wts\_as\_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.

- window the window size, in time units. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.
- wts an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.
- 1b\_time a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.
- na\_rm whether to remove NA, false by default.
- min\_df the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
- restart\_period the recompute period. because subtraction of elements can cause loss of precision, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.
- variable\_win if true, and the window is not a concrete number, the computation window becomes the time between lookback times.
- wts\_as\_delta if true and the time and time\_deltas are not given, but wts are given, we take wts as the time\_deltas.
- check\_wts a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

used_df	the number of degrees of freedom consumed, used in the denominator of the standard errors computation. These are subtracted from the number of observations.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

#### Details

Computes the correlation or covariance, or OLS regression coefficients and standard errors. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

#### Value

Typically a matrix, usually only one row of the output value. More specifically:

running\_covariance Returns a single column of the covariance of x and y.

**running\_correlation** Returns a single column of the correlation of x and y.

- **running\_covariance\_3** Returns three columns: the variance of x, the covariance of x and y, and the variance of y, in that order.
- running\_regression\_slope Returns a single column of the slope of the OLS regression.
- running\_regression\_intercept Returns a single column of the intercept of the OLS regression.
- running\_regression\_fit Returns two columns: the regression intercept and the regression slope of the OLS regression.
- **running\_regression\_diagnostics** Returns five columns: the regression intercept, the regression slope, the regression standard error, the standard error of the intercept, the standard error of the slope of the OLS regression.

#### **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \ldots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of *lookback times* are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The *j*th output is computed over indices *i* such that

$$b_j - W < t_i \leq b_j$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the *j*th output is computed over indices *i* such that

$$t_j - W + L < t_i \le t_j + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

# Examples

```
x <- rnorm(1e5)
y <- rnorm(1e5) + x
rho <- t_running_correlation(x, y, time=seq_along(x), window=100L)</pre>
```

t\_running\_sd3 Compute first K moments over a sliding time-based window

#### Description

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements over an infinite or finite sliding time based window, returning a matrix.

#### Usage

```
t_running_sd3(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_skew4(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
```

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```
check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_kurt5(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_sd(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_skew(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
```

```
na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_kurt(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_cent_moments(
  v,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 max_order_only = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
```

```
60
```

```
)
```

```
t_running_std_moments(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_cumulants(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
 variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
```

## Arguments

V	a vector of data.
time	an optional vector of the timestamps of v. If given, must be the same length as v. If not given, we try to infer it by summing the time_deltas.
time_deltas	an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts_as_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.

window	the window size, in time units. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, and variable_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
lb_time	a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this param- eter. Larger values mean fewer restarts and faster, though less accurate results.
variable_win	if true, and the window is not a concrete number, the computation window becomes the time between lookback times.
wts_as_delta	if true and the time_deltas are not given, but wts are given, we take wts as the time_deltas.
check_wts	a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.
normalize_wts	a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.
check_negative_	
	a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on

	every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.
max_order	the maximum order of the centered moment to be computed.
<pre>max_order_only</pre>	for running_cent_moments, if this flag is set, only compute the maximum order centered moment, and return in a vector.

# Details

Computes the number of elements, the mean, and the 2nd through kth centered (and typically standardized) moments, for k = 2, 3, 4. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

Given the length n vector x, we output matrix M where  $M_{i,j}$  is the order -j + 1 moment (*i.e.* excess kurtosis, skewness, standard deviation, mean or number of elements) of some elements  $x_i$  defined by the sliding time window. Barring NA or NaN, this is over a window of time width window.

#### Value

Typically a matrix, where the first columns are the kth, k-1th through 2nd standardized, centered moments, then a column of the mean, then a column of the number of (non-nan) elements in the input, with the following exceptions:

- t\_running\_cent\_moments Computes arbitrary order centered moments. When max\_order\_only is set, only a column of the maximum order centered moment is returned.
- t\_running\_std\_moments Computes arbitrary order standardized moments, then the standard deviation, the mean, and the count. There is not yet an option for max\_order\_only, but probably should be.
- **t\_running\_cumulants** Computes arbitrary order cumulants, and returns the kth, k-1th, through the second (which is the variance) cumulant, then the mean, and the count.

#### **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \ldots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of *lookback times* are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The *j*th output is computed over indices *i* such that

$$b_j - W < t_i \le b_j.$$

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the *j*th output is computed over indices *i* such that

$$t_j - W + L < t_i \le t_j + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

the kurtosis is *excess kurtosis*, with a 3 subtracted, and should be nearly zero for Gaussian input.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_ deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook. com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

#### t\_running\_sum

#### See Also

running\_sd3.

#### Examples

```
x <- rnorm(1e5)
xs3 <- t_running_sd3(x,time=seq_along(x),window=10)
xs4 <- t_running_skew4(x,time=seq_along(x),window=10)
# but what if you only cared about some middle values?
xs4 <- t_running_skew4(x,time=seq_along(x),lb_time=(length(x) / 2) + 0:10,window=20)</pre>
```

t\_running\_sum Compute sums or means over a sliding time window.

#### Description

Compute the mean or sum over an infinite or finite sliding time window, returning a vector the same size as the lookback times.

#### Usage

```
t_running_sum(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 10000L,
 variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE
)
t_running_mean(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 10000L,
```

```
variable_win = FALSE,
wts_as_delta = TRUE,
check_wts = FALSE
)
```

# Arguments

V	a vector.
time	an optional vector of the timestamps of v. If given, must be the same length as v. If not given, we try to infer it by summing the time_deltas.
time_deltas	an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts_as_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.
window	the window size, in time units. if given as finite integer or double, passed through. If NULL, NA_integer_, NA_real_ or Inf are given, and variable_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.
wts	an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na_rm is true. When there is no checking, an NA value will cause the output to be NA.
lb_time	a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.
na_rm	whether to remove NA, false by default.
min_df	the minimum df to return a value, otherwise NaN is returned, only for the means computation. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.
restart_period	the recompute period. because subtraction of elements can cause loss of preci- sion, the computation of moments is restarted periodically based on this parame- ter. Larger values mean fewer restarts and faster, though potentially less accurate results. Unlike in the computation of even order moments, loss of precision is unlikely to be disastrous, so the default value is rather large.
variable_win	if true, and the window is not a concrete number, the computation window be- comes the time between lookback times.
wts_as_delta	if true and the time and time_deltas are not given, but wts are given, we take wts as the time_deltas.

#### t\_running\_sum

check\_wts a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

#### Details

Computes the mean or sum of the elements, using a Kahan's Compensated Summation Algorithm, a numerically robust one-pass method.

Given the length n vector x, we output matrix M where  $M_{i,1}$  is the sum or mean of some elements  $x_i$  defined by the sliding time window. Barring NA or NaN, this is over a window of time width window.

# Value

A vector the same size as the lookback times.

#### **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \ldots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of *lookback times* are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The *j*th output is computed over indices *i* such that

$$b_j - W < t_i \leq b_j.$$

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the *j*th output is computed over indices *i* such that

$$t_j - W + L < t_i \le t_j + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{i-1} - W < t_i \le b_i.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined. The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/ http://people.xiph.org/~tterribe/notes/homs.html

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### Examples

```
x <- rnorm(1e5)
```

```
xs <- t_running_sum(x,time=seq_along(x),window=10)</pre>
```

xm <- t\_running\_mean(x,time=cumsum(runif(length(x))),window=7.3)</pre>

```
%-%, centcosums, centcosums-method
```

unconcatenate centcosums objects.

#### Description

Unconcatenate centcosums objects.

68

#### Note

# %-%

# Usage

## S4 method for signature 'centcosums,centcosums' x %-% y

# Arguments

х	a centcosums objects
У	a centcosums objects

# See Also

unjoin\_cent\_cosums

%-%

# unconcatenate centsums objects.

# Description

Unconcatenate centsums objects.

# Usage

х %-% у

## S4 method for signature 'centsums,centsums' x %-% y

# Arguments

х	a centsums objects
У	a centsums objects

# See Also

unjoin\_cent\_sums

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