

# Package ‘equaltestMI’

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**Type** Package

**Title** Examine Measurement Invariance via Equivalence Testing and Projection Method

**Version** 0.6.1

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**Description** Functions for examining measurement invariance via equivalence testing are included in this package. The traditionally used RMSEA (Root Mean Square Error of Approximation) cutoff values are adjusted based on simulation results. In addition, a projection-based method is implemented to test the equality of latent factor means across groups without assuming the equality of intercepts. For more information, see Yuan, K. H., & Chan, W. (2016) <[doi:10.1037/met0000080](https://doi.org/10.1037/met0000080)>, Deng, L., & Yuan, K. H. (2016) <[doi:10.1007/s11336-015-9491-8](https://doi.org/10.1007/s11336-015-9491-8)>, and Jiang, G., Mai, Y., & Yuan, K. H. (2017) <[doi:10.3389/fpsyg.2017.01823](https://doi.org/10.3389/fpsyg.2017.01823)>.

**License** GPL-3

**Encoding** UTF-8

**LazyData** true

**Depends** R (>= 3.1.0), lavaan, stats, utils

**Suggests** knitr,rmarkdown,MASS,semTools,printr

**BugReports** <https://github.com/gabriellajg/equaltestMI/issues>

**VignetteBuilder** knitr

**RoxygenNote** 7.1.1

**NeedsCompilation** no

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**Repository** CRAN

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eqMI.bootstrap	<i>Bootstrap procedure to test the equality of latent factor means using projection method</i>
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Description

Bootstrap procedure to test the equality of latent factor means using projection method

Usage

eqMI.bootstrap(..., B = 100, seed = 111)

Arguments

...	The same arguments as for any lavaan model. See lavaan::sem for more information.
B	The number of bootstrap samples. Default at 100.
seed	The initial seed to generate bootstrap samples. Default at 111.
bootstrap	If bootstrap resampling is used to obtain empirical p-value of the statistics.

Details

Perform bootstrap procedure when testing the equality of latent means using projection method. Note that raw data must be available for bootstrap resampling to be performed. With the projection method, the cross-group intercepts are not required to be the same for further tests. If bootstrap resampling is used, the test statistics are not referred to chi-squared distributions but to bootstrapped empirical distributions for significance testing. Percentage bootstrap critical values are calculated. This process might be time-consuming if the model is complex or the number of bootstrap samples (B) is large.

Value

bootstrap p-values of the tests of common and specific factors.

## References

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

## Examples

```
data(HolzingerSwineford)
semmodel<-'
L1 =~ V1 + V2 + V3
L2 =~ V4 + V5 + V6
L3 =~ V7 + V8
L4 =~ V9 + V10 + V11
'

run.bts <- eqMI.bootstrap(model = semmodel, data = HolzingerSwineford,
  group = "school", meanstructure = TRUE, B = 100, seed = 111)
```

---

eqMI.covtest

---

*Test the equality of two covariance matrices in population*


---

## Description

The first step of testing measurement invariance (MI) in multiple-group SEM analysis. The null hypothesis is tested using the method of Lagrange multipliers

## Usage

```
eqMI.covtest(..., lamb0 = NULL)
```

## Arguments

...	The same arguments as for any lavaan model. See <code>lavaan::sem</code> for more information.
lamb0	initial coefficients of Lagrange multiplier. If not pre-specified, 0.01 will be used.

## Details

The `eqMI.covtest` function is the first step to test MI. Under null hypothesis testing (NHT), a non-significant statistic is generally an overall endorsement of MI. If the null hypothesis is rejected then one may proceed to test other aspects of MI.

## Value

The likelihood ratio statistic, degrees of freedom, and p-value of the test.

## References

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

Yves Rosseel (2012). lavaan: An R Package for Structural Equation Modeling. *Journal of Statistical Software*, 48(2), 1-36. URL <http://www.jstatsoft.org/v48/i02/>.

## Examples

```
data(HolzingerSwineford)
semmodel<-'
L1 =~ V1 + V2 + V3
L2 =~ V4 + V5 + V6
L3 =~ V7 + V8
L4 =~ V9 + V10 + V11
'
cov.test <- eqMI.covtest(model = semmodel,
                        data = HolzingerSwineford,
                        group="school")
```

---

eqMI.main

---

*The main function to test measurement invariance*


---

## Description

Test measurement invariance with equivalence testing, projection methods, and adjusted RMSEA cutoffs for two groups.

## Usage

```
eqMI.main(
  ...,
  output = "both",
  equivalence.test = TRUE,
  adjRMSEA = TRUE,
  projection = FALSE,
  bootstrap = FALSE,
  quiet = TRUE,
  B = 100,
  seed = 111
)
```

## Arguments

...                    The same arguments as for any lavaan model. See `lavaan::sem` for more information.

	Users must explicitly specify the name of the input elements for this function to catch. For example, specify 'data = HolzingerSwineford' instead just 'HolzingerSwineford'.
output	If the function prints out results of covariance structure, mean structure, or both. The value of output must be mean, covariance, or both. When the tests involve mean structure (output = 'mean' or 'both'), both the strong and the strict tests of measurement invariance will be conducted.
equivalence.test	If equivalence.test=TRUE, equivalence testing is used for examining all statistics. RMSEA together with conventional or adjusted cutoff values will be used to gauge the goodness of fit.
adjRMSEA	If adjRMSEA=TRUE, adjusted RMSEA cutoff values are used for equivalence testing. See details in Yuan & Chan (2016).
projection	If projection=TRUE, projection method is used to test the equality of latent factor means. The advantage of the projection method over conventional multiple-group SEM approach is that the test of latent factor means can be conducted even when the equality of intercepts do not hold.
bootstrap	If bootstrap=TRUE, bootstrap is used to obtain empirical p-values for testing the equality of cross-group latent factor means.
quiet	If quiet=FALSE, a summary is printed out containing an overview of the different models that are fitted, together with some model comparison tests and fit measures. The results of equivalence testing will also be printed if equivalence testing is used. If quiet=TRUE (default), no summary is printed but results will be stored in 'AnnotatedOutput'.
B	The number of bootstrap samples used in bootstrap approach.
seed	The initial seed to generate bootstrap samples. Default at 111.

## Details

An all-in-one function with several added options to conduct a sequence of tests needed to evaluate MI. The chi-square statistics, except the one for testing the equality of covariance structure, are obtained based on `lavaan::sem` function. The test statistic of the covariance structure equality is obtained via the method of Lagrangian multiplier. Equivalence testing is enabled by setting `equivalence.test=TRUE` and this function will calculate T-size, RMSEA, and adjusted RMSEA cutoff values, and provide the goodness-of-fit.

## Value

A list is returned with:

- `AnnotatedOutput` Annotated output that will be printed to the console if `quiet==FALSE`.
- `eqMI.stat` Test statistics, degrees of freedom, p-values, ncp, T-sizes, RMSEAs, their cutoff values, and the goodness-of-fit under equivalence testing. A formatted version of `eqMI.stat` will be printed if `quiet=FALSE`.
- `convention.sem` Results of conventional multiple-group SEM using Lavaan. Returned object of `eqMI.semtest`.
- `projection.res` Results of projection methods on tests of latent means. Returned object of `eqMI.projection` and `eqMI.bootstrap`.

## References

- Deng, L., & Yuan, K. H. (2016). Comparing Latent Means Without Mean Structure Models: A Projection-Based Approach. *Psychometrika*, 81(3), 802-829. <https://doi.org/10.1007/s11336-015-9491-8>
- Jiang, G., Mai, Y., & Yuan, K. H. (2017). Advances in Measurement Invariance and Mean Comparison of Latent Variables: Equivalence Testing and A Projection-Based Approach. *Frontiers in Psychology*, 8, 1823.
- Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426. <https://doi.org/10.1037/met0000080>

## Examples

```
data(HolzingerSwineford)
semmodel<-'
L1 =~ V1 + V2 + V3
L2 =~ V4 + V5 + V6
L3 =~ V7 + V8
L4 =~ V9 + V10 + V11
'

# If raw data are available;

test <- eqMI.main(model = semmodel, data = HolzingerSwineford,
  group = "school", meanstructure = TRUE,
  output = 'both', quiet = FALSE,
  equivalence.test = TRUE, adjRMSEA = TRUE,
  projection = TRUE, bootstrap = FALSE)

# when only sample statistics are available;
# sample.cov need to be provided for tests of covariance structure;
# sample.mean need to be provided for tests of mean structure;

school1 <- subset(HolzingerSwineford, school==1)[-12]
school2 <- subset(HolzingerSwineford, school==2)[-12]
test <- eqMI.main(model = semmodel,
  sample.nobs = c(nrow(school1), nrow(school2)),
  sample.cov = list(cov(school1), cov(school2)),
  sample.mean = list(colMeans(school1), colMeans(school2)),
  meanstructure = TRUE, output = 'both', quiet = FALSE,
  equivalence.test = TRUE, adjRMSEA = TRUE,
  projection = TRUE, bootstrap = FALSE)
```

---

eqMI.ncp

---

*Obtain noncentrality parameter of a chisquare distribution*


---

## Description

Calculate the noncentrality parameter as well as the model misspecification  $\epsilon_t$  given its lower-tail critical value.

**Usage**

```
eqMI.ncp(T, df, N, m, alpha = 0.05)
```

**Arguments**

T	A chi-square statistic
df	Degrees of freedom
N	Total sample size of all groups
m	Number of groups
alpha	Significance level. Default at 0.05.

**Details**

This function is to compute the noncentrality parameter `ncp`, the model misspecification `epsilon_t`, and its corresponding `RMSEA_t`. With equivalence testing, the model misspecification is also the minimum tolerable size that a researcher needs to tolerate if one wishes to proceed with further restricted tests. The formula from Venables (1975) is used for obtaining the noncentrality parameter of a non-central chi-square distribution given its lower-tail critical value.

**Value**

The noncentrality parameter `ncp`, the minimum tolerable size `epsilon_t`, and `RMSEA_t` under equivalence testing.

**References**

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

**Examples**

```
alpha <- .05
n_1 <- 200
n_2 <- 200
N <- n_1 + n_2
m <- 2
# A made-up likelihood-ratio statistic
T_ml <- 8.824
df <- 6
ncp <- eqMI.ncp(T = T_ml, df = df, N = N, m = m, alpha = alpha)
```

eqMI.projection

*Projection-based method for testing latent means equality***Description**

Perform projection method for testing the equality of latent means without requiring the equality of cross-group intercepts to hold.

**Usage**

```
eqMI.projection(...)
```

**Arguments**

... The same arguments as for any lavaan model. See `lavaan::sem` for more information. All models fitted by Lavaan are estimated by fixing the variances of latent factors to 1.

Users must explicitly specify the name of the input elements for this function to catch. For example, specify `'data = HolzingerSwineford'` instead just `'HolzingerSwineford'`.

**Details**

Perform projection method for testing the equality of two latent means without requiring the cross-group intercepts to be the same. A validity index is provided as the proportion of the differences in manifest variables intercepts explained by latent mean differences as a gauge of the quality of measurements.

**Value**

A list is returned with:

`fit.metric` test of metric invariance (factor loadings). This is a prerequisite for testing equality of latent means.

`mvdif.test` t tests of the cross-group sample means for each variable.

`chi.stat` Three chi-square tests for intercepts, common factors, and specific factors. `chi.stat` will be needed for equivalence testing.

`common.test` t tests of common factors for each variable.

`specific.test` t tests of specific factors for each variable.

`latent.test` t tests of latent means

`V.index` validity index

`Pmat` projection matrix of intercepts into the space of common factors

`Qmat` projection matrix of intercepts into the space of specific factors

## References

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

## Examples

```
data(HolzingerSwineford)
semmodel<-'
L1 =~ V1 + V2 + V3
L2 =~ V4 + V5 + V6
L3 =~ V7 + V8
L4 =~ V9 + V10 + V11
'

run.proj <- eqMI.projection(model = semmodel, data = HolzingerSwineford,
                             group = "school", meanstructure = TRUE)
```

---

eqMI.RMSEA

---

*Obtain the RMSEA cutoff values for equivalence testing*


---

## Description

Generate adjusted cutoff values of RMSEA for equivalence testing corresponding to conventional cutoff values .01, .05, .08, and .10.

## Usage

```
eqMI.RMSEA(N, m, df)
```

## Arguments

N	Total sample size of all groups
m	Number of groups
df	Degree of freedom

## Details

The adjusted cutoff values of RMSEA for equivalence testing can be obtained with N, m, df and transformed variables. Formulas are estimated using simulation studies and the coefficients are given in Table 11 of the reference.

## Value

The adjusted cutoff values corresponding to conventional cutoff values .01, .05, .08, and .10.

References

Steiger, J. H. (1980). Statistically based tests for the number of common factors. In the annual meeting of the Psychometric Society. Iowa City, IA.

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

Examples

```
alpha <- .05;
N <- 200;
m <- 1;
T_m1 <- 28.446; #the statistic T_m1 for group 1;
df <- 24;
eqMI.RMSEA(N = N, m = m, df = df);
```

---

eqMI.semtest	<i>Measurement invariance tests using lavaan</i>
--------------	--

---

Description

Conventional multiple-group SEM to test measurement invariance. A sequence of chi-squared and chi-squared difference tests will be conducted.

Usage

```
eqMI.semtest(..., output = "both", quiet = FALSE)
```

Arguments

...	The same arguments as for any lavaan model. See <code>lavaan::cfa</code> and <code>lavaan::lavOptions</code> for more information about the arguments.  Users must explicitly specify the name of the input elements for this function to catch. For example, specify <code>'eqMI.semtest(model = semmodel, data = HolzingerSwineford)'</code> instead just <code>'eqMI.semtest(semmodel, HolzingerSwineford)'</code> .
output	If the function prints out results of covariance structure, mean structure, or both. The value of output must be mean, covariance, or both. When the tests involve mean structure (output = 'mean' or 'both'), both the strong and the strict tests of measurement invariance will be conducted.
quiet	If quiet=FALSE (default), a summary is printed out containing an overview of the different models that are fitted, together with some model comparison tests. If quiet=TRUE, no summary is printed but results will be stored in the object.

## Details

This is a wrapper around the (now deprecated) `measurementInvariance` in package `semTools`, with the following default options: `std.lv = FALSE`, `fit.measures = "default"`, and `method = "satorra.bentler.2001"`. See `semTools` for more information. This function is now updated to `semTools::measEq.syntax`.

## Value

A list is returned with:

`LavaanOut` A sublist in `convention.sem`. Contains lavann style output and results for each chi-squared and chi-squared difference.

`Mean.part` A sublist in `convention.sem`. Contains test statistics and fit measures on invariance tests of mean structure.

`Cov.part` A sublist in `convention.sem`. Contains test statistics and fit measures on invariance tests of covariacne structure.

## Author(s)

The maintainer, Ge Jiang, adapted the original source code of `measurementInvariance()` in the **lavaan** and **semTools** packages written by Yves Rosseel, Sunthud Pornprasertmanit, and Terrence D. Jorgensen (permission obtained).

## References

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

Yves Rosseel (2012). *lavaan: An R Package for Structural Equation Modeling*. *Journal of Statistical Software*, 48(2), 1-36.

semTools Contributors. (2016). *semTools: Useful tools for structural equation modeling*. R package version 0.4-14. Retrieved from <https://CRAN.R-project.org/package=semTools>

## See Also

[sem](#), [measurementInvariance](#)

## Examples

```
data(HolzingerSwineford)
semmodel<- '
L1 =~ V1 + V2 + V3
L2 =~ V4 + V5 + V6
L3 =~ V7 + V8
L4 =~ V9 + V10 + V11
'

run.sem <- eqMI.semtest(model = semmodel, data = HolzingerSwineford,
  group = "school", meanstructure = TRUE)
```

---

HolzingerSwineford	<i>Holzinger and Swineford (1939) cognitive tests data in 301 children from two schools</i>
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---

**Description**

This classic data set contains reported data on cognitive tests from two elementary schools.

**Usage**

```
data(HolzingerSwineford)
```

**Format**

A data frame with 301 observations on 11 numeric variables and a group indicator (12 variables in total).

**Details**

11 variables used in Deng and Yuan (2016) are included in this version of data, they are: visual perception, cubes, paper form board, paragraph comprehension, word classification, word meaning, counting dots, straight-curved capitals, deduction, problem reasoning, and series completion.

This data set with more variables are available in other R packages such as lavaan, and OpenMX.

**References**

Deng, L., & Yuan, K. H. (2016). Comparing Latent Means Without Mean Structure Models: A Projection-Based Approach. *Psychometrika*, 81(3), 802-829.

Yuan, K. H., & Chan, W. (2016). Measurement invariance via multigroup SEM: Issues and solutions with chi-square-difference tests. *Psychological methods*, 21(3), 405-426.

**See Also**

[HolzingerSwineford1939](#), [HS.ability.data](#)

**Examples**

```
data(HolzingerSwineford)
```

---

LeeAlOtaiba	<i>Lee and Al Otaiba (2015) early literacy skills in four socioeconomic groups</i>
-------------	--

---

### Description

This data set contains means and covariance matrices of early literacy skills measured in four different sociodemographic groups. The results are reported in Table 1 of Lee and Al Otaiba (2015).

### Usage

```
data(LeeAlOtaiba)
```

### Format

A list of four data frames. Every data frame is of dimension 7 by 6. The first row contains sample means of the six variables. The next six rows contain sample covariance matrix. The column names of the data frame are the names of the six variables. The row names of the data frame contain a label 'mean' and the names of the six variables.

### Details

Six variables used in Lee and Al Otaiba (2015) to measure literacy constructs, including (1) letter-name fluency, (2) letter-sound fluency, (3) blending, (4) elision, (5) real words spelling, and (6) pseudo-words spelling.

Following from Snow's (2006) definition of componential skills and the work of Schatschneider, Fletcher, Francis, Carlson, and Foorman (2004) on National Early Literacy Panel (NELP), the six variables aim to measure three aspects of literacy constructs: (1) alphabet knowledge, (2) phonological awareness, and (3) spelling.

The four sociodemographic groups are: (1) boys who are ineligible for FRL (n=78); (2) boys who are eligible for FRL (n=65); (3) girls who are ineligible for FRL (n=175); and (4) girls who are eligible for FRL (n=165).

### References

Lee, J. A. C., & Al Otaiba, S. (2015). Socioeconomic and gender group differences in early literacy skills: A multiple-group confirmatory factor analysis approach. *Educational Research and Evaluation*, 21(1), 40–59. <https://doi.org/10.1080/13803611.2015.1010545>

### Examples

```
data(LeeAlOtaiba)
# If one wants to extract the two groups used in the paper:
Group1 <- LeeAlOtaiba[[1]]
Group2 <- LeeAlOtaiba[[2]]
```

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